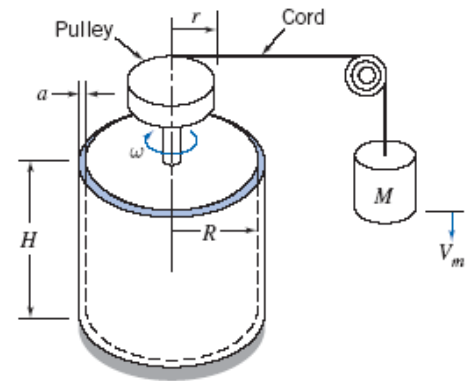


## Problem 2.60

[Difficulty: 2]

**2.60** A concentric cylinder viscometer is driven by a falling mass  $M$  connected by a cord and pulley to the inner cylinder, as shown. The liquid to be tested fills the annular gap of width  $a$  and height  $H$ . After a brief starting transient, the mass falls at constant speed  $V_m$ . Develop an algebraic expression for the viscosity of the liquid in the device in terms of  $M$ ,  $g$ ,  $V_m$ ,  $r$ ,  $R$ ,  $a$ , and  $H$ . Evaluate the viscosity of the liquid using:



$$M = 0.10 \text{ kg}$$

$$r = 25 \text{ mm}$$

$$R = 50 \text{ mm}$$

$$a = 0.20 \text{ mm}$$

$$H = 80 \text{ mm}$$

$$V_m = 30 \text{ mm/s}$$

Solution: Apply Newton's law of viscosity.

Basic equations:  $\tau = \mu \frac{du}{dy}$      $\Sigma M = 0$      $T = \tau AR$

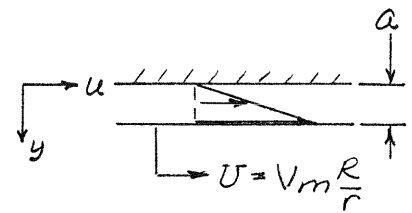
Assumptions: (1) Newtonian liquid  
(2) Narrow gap, so linear velocity profile  
(3) Steady angular speed

Summing torques on the rotor

$$\Sigma M = Mgr - \tau AR = I \alpha = 0 \quad ; \quad A = 2\pi RH$$

Because  $a \ll R$ , treat the gap as plane. Then

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} = \mu \frac{U - 0}{a - 0} = \mu \frac{U}{a} = \frac{\mu V_m R}{ar}$$



Substituting,

$$Mgr - \frac{\mu V_m R}{ar} 2\pi RH R = Mgr - \frac{2\pi \mu V_m R^3 H}{ar} = 0$$

so

$$\mu = \frac{Mgr^2 a}{2\pi V_m R^3 H}$$

Evaluating for the given data

$$\mu = \frac{1}{2\pi} \times 0.10 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \times (0.025)^2 \text{ m}^2 \times 0.0002 \text{ m} \times \frac{\text{s}}{0.030 \text{ m}} \times \frac{1}{(0.050)^3 \text{ m}^3} \times \frac{1}{0.080 \text{ m}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$$\mu = 0.0651 \text{ N} \cdot \text{s} / \text{m}^2 \quad (65.1 \text{ mPa} \cdot \text{s})$$